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(No) Eternal inflation and precision Higgs physics

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Abstract: Even if nothing but a light Higgs is observed at the LHC, suggesting that the Standard Model is unmodified up to scales far above the weak scale, Higgs physics can yield surprises of fundamental significance for cosmology. As has long been known, the Standard Model vacuum may be metastable for low enough Higgs mass, but a specific value of the decay rate holds special significance: for a very narrow window of parameters, our Universe has not yet decayed but the current inflationary period can not be future eternal. Determining whether we are in this window requires exquisite but achievable experimental precision, with a measurement of the Higgs mass to 0.1 GeV at the LHC, the top mass to 60 MeV at a linear collider, as well as an improved determination of α_s by an order of magnitude on the lattice. If the parameters are observed to lie in this special range, particle physics will establish that the future of our Universe is a global big crunch, without harboring pockets of eternal inflation, strongly suggesting that eternal inflation is censored by the fundamental theory. This conclusion could be drawn even more sharply if metastability with the appropriate decay rate is found in the MSSM, where the physics governing the instability can be directly probed at the TeV scale.

Keywords: Space-Time Symmetries, Cosmology of Theories beyond the SM, Higgs Physics, Standard Model.

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1. Introduction

The most remarkable recent discovery in fundamental physics is that the Universe is accelerating [1 − 3]. Will this acceleration last forever or eventually terminate? What will be the global picture of space-time, and is this a meaningful question? These are some of the most pressing theoretical issues of our time. The idea of eternal inflation [4, 5] appears to be supported by the existence of a landscape of metastable vacua in the string theory [6, 7]. It provides a plausible mechanism to populate the different vacua making it possible to apply environmental arguments to explain the smallness of the cosmological constant [8]. However, the necessity to talk about causally disconnected space-time regions to describe eternal inflation makes it extremely challenging to implement this picture in a full theory of quantum gravity. At the moment we do not even know if there are any sharply-defined observables for such a theory to compute; if any exist, they likely reside at future boundaries, with no obvious connection to our own observations (see [9 − 12] for some of attempts to make sense of this picture). This problem and the analogy of the emerging causal structure with that of the evaporating black hole, where a naive semiclassical logic fails by predicting the information loss, naturally suggests that semiclassical physics may also be misleading in this case, and a qualitatively new approach is needed to describe the eternally inflating Universe (see, e.g., [13] for a recent discussion). Clearly, in this situation any experimental data shedding some light on the nature of the cosmological acceleration is extremely welcome.

At the classical level the choice is very simple. Either we live in a local minimum of energy and the cosmic acceleration will last forever, or the acceleration is driven by the potential energy of the slowly rolling scalar field and may eventually terminate. Things change qualitatively at the quantum level. In the slow roll case the Universe can still be eternally inflating if the acceleration parameter is sufficiently close to being constant [13],

$$\frac{\dot{H}_\Lambda}{H_\Lambda^4} \lesssim M_{Pl}^{-2} , \tag{1.1}$$

where H_Λ is the Hubble expansion rate corresponding to the current vacuum energy. Numerically, if the bound (1.1) does not hold the current stage of the cosmological acceleration may last as long as $\sim 10^{130}$ years without being eternal. Current data show that the equation of state of the negative energy component is rather close to that of the vacuum energy, $\dot{H}_\Lambda/H_\Lambda^2 \lesssim \text{few} \cdot 10^{-2}$. The bound (1.1) implies that the next theoretically meaningful value for this ratio is at the level $H_\Lambda^2/M_{Pl}^2 \sim 10^{-120}$. This indicates that in the slow roll case the chances to test the eternal nature of the cosmological acceleration by directly measuring the time dependence of the acceleration parameter $\dot{H}_\Lambda/H_\Lambda^2$ are not high in the next 10^{130} years or so.

The situation is more interesting if we live in a local energy minimum. In the string landscape our positive energy vacuum is inevitably metastable towards decay into a lower energy state [14]. Given the extreme smallness of the vacuum energy this decay is likely to proceed into a rapidly crunching negative energy Universe. The decay goes through the non-perturbative nucleation of bubbles of the new vacuum that afterwards expand with the speed of light. Whether or not the current stage of the cosmological acceleration is eternal depends on the rate Γ of the bubble nucleation. If the decay rate is fast enough

$$\Gamma \geq \Gamma_{\rm cr} = \frac{9}{4\pi} H_\Lambda^4 , \tag{1.2}$$

bubbles collide and eventually all the Universe suffers a transition into the new vacuum. The current stage of inflation is not eternal in this case. On the other hand, for slower decay rates the expansion of the Universe wins and bubbles never fill the whole inflating volume. Of course, bubble walls propagate with the speed of light so we have no chances of observing bubbles without being immediately killed by a domain wall. However, in principle, we can establish whether the bound (1.2) holds by performing high precision measurements of the underlying particle physics parameters, so that it becomes possible to identify a nearby negative energy minimum and to calculate a lower bound for the decay rate with high enough precision. We say 'a lower bound' because there are always potential instabilities coming from high energy physics we cannot control, but it suffices to find fast enough infrared calculable instabilities to see that inflation cannot be eternal.

High precision measurements are needed since, given that the Universe did not decay up to the present we know that the decay rate Γ cannot be significantly faster than the lower bound (1.2). Imposing that the probability (p) not to decay up to now is larger than, say, 5% (2σ) gives $\Gamma \leq 52.3\,\Gamma_{\rm cr}$, for generic p the rate will be in the window

$$\Gamma_{\rm cr} \leq \Gamma \leq 17.45 \, \log\left(p^{-1}\right) \Gamma_{\rm cr} \, . \tag{1.3}$$

Given that Γ depends exponentially on the underlying particle physics parameters, we will see that even a change in the value of Γ by a factor of 50 corresponds to a tiny change of particle masses and coupling constants.

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In this paper we argue that, with a certain amount of luck, challenging but feasible measurements at the TeV scale may establish the existence of a fast enough instability, such that the bound (1.2) holds. Our main focus will be a minimal scenario of this kind, which is realized if there is no new physics up to very high energies of order the GUT scale. As reviewed in section 2, in this case if the Higgs mass is light enough, the Standard Model (SM) potential develops a negative energy minimum at large values of the Higgs field. We show then that one can verify whether the bound (1.2) holds for this particular decay channel by performing high precision measurements of the Higgs mass (with uncertainty $\Delta m_H \sim 0.2\,\mathrm{GeV}$), the top mass ($\Delta m_t \sim 60\,\mathrm{MeV}$) and the strong coupling constant ($\Delta\alpha_s/\alpha_s \sim 10^{-3}$). These numbers are achievable with the future data from the LHC and a linear collider. On the theoretical side one will need to improve the existing calculations of the decay rate by including at least one extra electroweak and two strong loops.

Of course, we will never be able to directly verify that the Standard Model holds up to the GUT scale. However, the window for the SM parameters allowing the decay rate to be fast enough, such that the current cosmological acceleration is not eternal, is extremely narrow. Consequently, if no new physics is found at the LHC and the parameters turn out to be in that window, it is reasonable to take this scenario seriously and to try to find an underlying reason why Nature worked so hard to avoid eternal inflation. We outline reasons why Nature might want to censor eternal inflation in section 3. We stress that we do not necessarily find these arguments plausible — oNe of the AutHors doesn't believe them — they certainly require what looks like a conspiratorial restriction on particle physics models, (although milder examples of surprising gravitational limitations on effective field theories have been seen in other contexts [15 – 17]). It seems much more reasonable that eternal inflation *does* occur, and that we have to learn how to properly deal with it. Nonetheless the counter arguments are not unreasonable, and it is interesting that entirely feasible experimental observations could strongly push us to take them seriously.

These arguments *predict* that the vacuum decay rate is within the region of eq. (1.3). *A priori*, there is no reason to expect this region to be so narrow. However it is very narrow in the real world. This is nothing but a reflection of the famous cosmic coincidence problem (i.e. why $T_{\mathrm{U}} \sim H_\Lambda^{-1}$). At the moment, the best solution to this problem is provided by the Weinberg's argument [8]. However, we would like to stress that the fact that the non-eternal inflation bound predicts a very narrow range in the Standard Model parameters is independent of whether the Weinberg's explanation is the correct one and is just a consequence of experimental data. On a similar note, observing the Standard Model parameters in this tiny window would neither support nor disfavor Weinberg's argument.

The situation can be even more interesting if new physics is discovered at the LHC, as in this case a negative energy minimum could be found at directly accessible energy scales. In section 2.1 we briefly review one scenario of this kind, which is supersymmetry with large soft-breaking trilinear couplings. We conclude by summarizing our results and outlining future directions.

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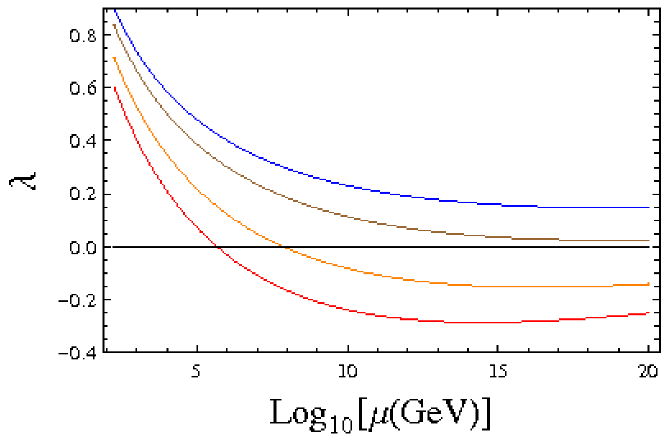

Figure 1: Running of the Higgs quartic coupling $\lambda(\mu)$ for $m_t = 170.9\,\text{GeV}$ for different Higgs masses (from below) $m_H = 110$, 120, 130 and 135 GeV (see appendix for formulæ and notations). For low masses λ turn negative at high scales μ and the SM vacuum become unstable.

2. The standard model analysis

In the case the LHC will discover nothing else but the Higgs up to the TeV scale, it is reasonable to assume (as we will do in this section) that the Standard Model may hold up to very large scales, say the seesaw, the GUT or the Plank scales. In this scenario it is well known [18]–[35] that the Higgs potential may develop an instability that makes our vacuum decay. This is due to the fact that, depending on the physical Higgs and top masses, the running may turn the quartic coupling, and thus the potential, negative at high scales (see figure 1). Requiring our vacuum to be sufficiently long lived with respect to the present age of the Universe (T_{U}) a lower bound on the Higgs mass has been derived [34], giving[1]

$$m_H \gtrsim 111 \text{ GeV}\,. \tag{2.1}$$

This decay channel disappears when $m_H \gtrsim 129\,\text{GeV}$.

If we assume that the current observed acceleration of the Universe is due to the vacuum energy (as opposed to some quintessence field) we are led to the following fundamental question: *will our Universe eternally inflate or not?* As we will show below there is a critical value for the Higgs mass such that if the Higgs is lighter, we will be able to say that our Universe will not inflate forever. Inflation in a false vacuum (with a positive cosmological constant $\Lambda = 3H_\Lambda^2/(8\pi G_N)$) may or may not be eternal depending on whether the decay rate Γ is smaller or larger than a critical value. Indeed the inflating volume as a function of time is

$$V_{\text{infl}} = V_0\, e^{3H_\Lambda t} e^{-\Gamma\, \widehat{\text{Vol}}_4(t)}, \tag{2.2}$$

where $\widehat{\text{Vol}}_4(t) = (4\pi/3H_\Lambda^3)\, t$ is the 4-volume spanned by the past light-cone of a comoving point after a time t [36], the first factor is due to the de-Sitter expansion while the second

[1]Using the updated values for the top mass and the strong coupling constant given below.

represents the exponential decay due to bubble nucleation. When $\varepsilon \equiv \Gamma/H_\Lambda^4 < 9/4\pi$ the expansion rate of the Universe wins against the decay rate, the inflationary volume expands exponentially and the Universe eternally inflates. In the opposite case bubbles percolate and inflation ends globally in a finite time.

Notice that just requiring that bubbles percolate is not enough to guarantee that inflation ends globally; indeed in [36] the critical value for percolation has been identified to lie in the interval

$$1.1 \cdot 10^{-6} < \varepsilon < \frac{9}{4\pi} n_c = 0.24\,, \tag{2.3}$$

which is smaller than $9/4\pi$ (here $n_c = 0.34$ is the critical ratio between the volume in the bubbles and the total volume). In what follows, we are being somewhat sloppy in the terminology and by percolation transition we refer to the transition to the non-eternal regime, $\varepsilon = 9/4\pi$.

In order to translate the bound $\Gamma/H_\Lambda^4 > \varepsilon$ into a bound for the Higgs mass we need to know how Γ depends on m_H. The expression for the decay rate per unit space-time volume is [37 − 39]

$$\Gamma = \frac{1}{V_4} e^{-S_E}\,, \tag{2.4}$$

where S_E is the euclidean action calculated on the bounce solution, $V_4 = R^4$ and R is the size of the bubble that maximizes the rate [34]. At one loop S_E reads

$$S_E = \frac{16\pi^2}{|\lambda(\mu)|} + \Delta S(\mu R)\,, \tag{2.5}$$

where $\lambda(\mu)$ is the Higgs quartic coupling at the scale $\mu \simeq R^{-1}$ and $\Delta S(\mu R)$ represents the finite and the not log-enhanced one-loop corrections, which have been computed in [34]. This expression can be improved by including the two-loop log-enhanced corrections through resumming up to two loops the RGE evolution of the couplings [25, 27] from the weak scale to the scale of the bubble R^{-1}. Requiring that $\Gamma > H_\Lambda^4 \varepsilon$ gives an upper bound on S_E; in turn this gives a lower bound to $|\lambda(\mu \simeq 1/R)|$. Since $\lambda(\mu \simeq 1/R)$ is negative this finally gives an upper bound to the quartic coupling at the weak scale, thus to the Higgs mass.

Before entering the details of the calculation of Γ, we can anticipate some features of the result. The upper bound from not being eternally inflating will be very close to the stability lower bound (2.1) because the dependence on T_U of the latter is logarithmic and, as it happens in our Universe, $T_{\rm U} \sim H_\Lambda^{-1}$. The precision required in order to determine whether we are not eternally inflating will be set by the difference between the (would be) observed Higgs mass and the one saturating the no eternal inflation bound. We can estimate an upper bound to this difference by imposing the probability that our Universe has not decayed yet to be larger than p, i.e.

$$e^{-\mathrm{Vol}_4(T_U)\,\Gamma} > p\,, \tag{2.6}$$

and plugging $\mathrm{Vol}_4(T_U) \simeq 0.08 \cdot H_\Lambda^{-4}$ for the space-time volume in the past lightcone of an observer at time T_U. As we will see momentarily (see eq. (2.8) below), in this way one obtains a lower bound on the Higgs mass that differs from the percolation bound by

$$\Delta m_H(\text{GeV}) = 0.05 \log \left(\frac{9}{4\pi} \frac{0.08}{\log p^{-1}} \right) , \tag{2.7}$$

which gives 0.2 GeV and 0.15 GeV for $p = 0.05$ ($\sim 2\sigma$) and $p = 0.32$ ($\sim 1\sigma$) respectively. This means that if the Higgs potential prevents the Universe from being eternally inflating, the measured Higgs mass must lie in the tiny window within 0.2 GeV below the percolation bound.

For the same reason the radius of the bubble will be close to that found in [34], i.e. $R^{-1} \sim 10^{17}$ GeV. The reason for such a small scale is that an optimal bubble radius is set as a result of a competition of different logarithmic effects — the top contribution to the quartic coupling running making it negative, and the gauge contributions to the running pushing the quartic coupling up.

Given that high precision is needed and since we have to run for a huge interval of scales we want to compute the rate with the best possible accuracy. In order to perform the running of $\lambda(\mu)$, as done in [27] we integrated the two-loop RGE equations of the three gauge couplings, the top Yukawa and the quartic couplings. This corresponds to including the contributions at the following orders: $\mathcal{O}(\alpha_W^2)$, $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha_W \alpha_s)$, where α_W stays for both the electroweak and the top Yukawa couplings. We also included three-loop $\mathcal{O}(\alpha_s^3)$ contributions (for the top Yukawa [40, 41] and the strong coupling constant [42, 43]) to the running because they are known and are comparable to those of order $\mathcal{O}(\alpha_W \alpha_s)$ (since $\mathcal{O}(\alpha_W) \sim \mathcal{O}(\alpha_s^2)$). At this order we also need the initial condition for the top Yukawa and quartic coupling matched at one-loop for the Higgs pole mass [44] and up to one-loop weak [45] and two-loop strong [46 – 48] for the top pole mass. $\mathcal{O}(\alpha_s^4)$ corrections to the α_s running [49, 50], three-loop strong [51] and mixed two-loop strong/weak [52] contributions for the top mass are also known and we used them to estimate higher order corrections.

Using the preliminary value for the top mass in [53] $m_t = 172.6 \pm 0.8_{\text{stat}} \pm 1.1_{\text{syst}} \simeq 172.6 \pm 1.4$ GeV, and the current value [54] of the strong coupling $\alpha_s = 0.1176 \pm 0.0020$ we get the no-eternal inflation bound (see the appendix for the details of the calculation)

$$m_H(\text{GeV}) < 109.1 + 4.4 \times \frac{m_t(\text{GeV}) - 172.6}{1.4} - 2.5 \times \frac{\alpha_s(m_Z) - 0.1176}{0.0020} + 0.05 \log \left(\frac{9}{4\pi\varepsilon} \right) \pm 3_{\text{th}} , \tag{2.8}$$

with bubbles of size $R^{-1} \simeq 10^{16}$ GeV for Higgs masses saturating the bound. The theoretical error in eq. (2.8) is only indicative: we estimated it by including the effects of the known higher order corrections in the matching of the top mass, the running of α_s and by varying the matching scale[2] μ between m_Z and $v = 246.22$ GeV. All these methods give a shift to the bound (2.8) of about 3 GeV or less. As usual the estimation of the theoretical errors is very uncertain and should be taken at the level of order of magnitude.

Since the direct search for the Higgs at LEP2 [55] gives a lower bound $m_H > 114.4$ GeV at 95% CL, which is within 1σ from (2.8), the discovery of a light Higgs may signal that we

[2]For the central value of the bound in eq. (2.8) we matched the top Yukawa at $\mu = m_t$, the Higgs quartic coupling at $\mu = m_H$ and the gauge couplings at $\mu = m_Z$, see the appendix for more details.

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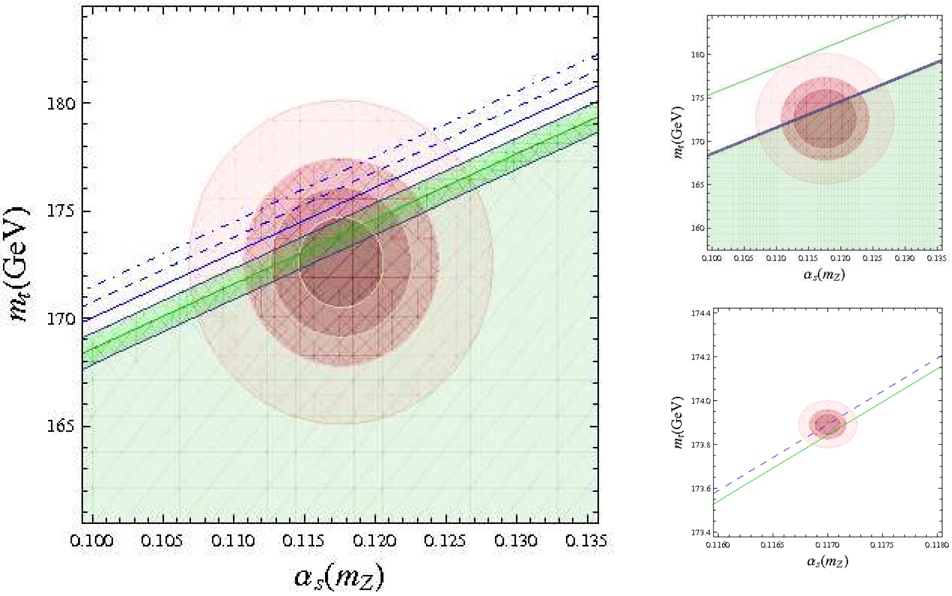

Figure 2: *Left:* measured $m_t(GeV)$ and $\alpha_s(m_Z)$ at 1, 2, 3 and 5σ (ellipses) *vs* the percolation bound for different Higgs masses (blue lines). The line in the middle of the shaded strip corresponds to the LEP2 bound $m_H = 114.4\,\text{GeV}$ (the excluded experimental region is green shaded), all the other lines are at step of 3 GeV in the Higgs mass. The shaded strip corresponds to the $\pm 3\,\text{GeV}$ theoretical error on the bound. This shows that, with the current experimental and theoretical errors, non-eternal inflation is excluded at less than 1σ level, i.e. it is compatible with current observations. *Right up:* Rule out scenario: all the values stay the same and an heavy Higgs mass is observed ($m_H \simeq 145\,\text{GeV}$), eternal inflation is not excluded by the bound; *Right down:* Rule in scenario: both the experimental and the theoretical errors have been substantially decreased and the central values moved such that the bound on the Higgs mass (dashed blue line) ($m_H = 115.2\,\text{GeV}$ in the figure) is above the experimental value (green line, $m_H = 115\,\text{GeV}$ in this case) by $\sim 2\sigma$; the Universe does not experience eternal inflation.

are not eternally inflating. In this case in order to be sure (see figure 2) one should reduce the theoretical and experimental uncertainties and check whether the Higgs mass is above or below the bound (2.8). After LHC the scenario will be reversed; while the Higgs mass will be known with high accuracy, the precision on the top mass will not improve much, at this point it will become more useful to rewrite the bound (2.8) as a bound on the top mass:

$$m_t(\text{GeV}) > 174.4 + 0.3 \times (m_H(\text{GeV}) - 115) + 0.8 \times \frac{\alpha_s(m_Z) - 0.1176}{0.0020} \pm 1_{\text{th}}\,. \qquad (2.9)$$

Note that in this case also the size of the bubble shrinks. For the choice of the parameters such that the percolation bound is saturated at $m_H \sim 115\,\text{GeV}$, the size is $R^{-1} \sim 10^{17}\,\text{GeV}$.

On the contrary if the Higgs turns out to be heavier than the bound (2.8) we will not be able to come to a definite conclusion on the fate of the Universe. Indeed, the rate Γ can always receive contributions from unknown physics (even at the Planck scale) that can make the decay rate large enough to avoid eternal inflation. Therefore our bound, together with the assumption that the running is not affected by new physics up to large enough scales, allows us to determine the future evolution of the Universe only in the case the Higgs mass satisfies eq. (2.8).

Using the bound (2.8) it is straightforward to see that the range of Γ where we are not eternally inflating and the probability that our Universe has not decayed yet is larger than 5%, correspond to a tiny window for the Higgs mass within 0.2 GeV below the bound (2.8). In order to verify the no eternal inflation hypothesis all the uncertainties in eq. (2.8), i.e. those in the top and the Higgs masses, α_s and the theoretical uncertainties must be reduced at this level. This is a challenging task both experimentally and theoretically.

On the experimental side one would need to reduce the error on the top mass by a factor of ~ 20, the one on α_s by one order of magnitude and to measure the Higgs mass with an error less than a fifth of GeV. The latter should be within the reach of LHC [56]. It should also be able to increase somewhat the precision on m_t, although the required precision on the top mass will be probably achieved only at a linear collider. On the other hand, the most precise determination of α_s at the moment comes from lattice simulations and the situation does not seem to change with the forthcoming experiments. Hopefully numerical calculations will be able to increase their power (at the same time reducing systematic uncertainties) by an order of magnitude on the decades time-scale.

On the theoretical side, both the running and the matching conditions for gauge couplings and masses should be improved at least by one extra electroweak loop and two strong loops. This also implies an extra loop in the calculation of the bouncing solution and the corrections to the corresponding action in the decay rate expression.

Our working assumption so far was that the physics up to the scale of the bounce, $R^{-1} \simeq 10^{17}\,\text{GeV}$, is exactly that of the Standard Model. However, there is one set of corrections that we definitely cannot ignore — those coming from gravity. Given the relatively small size of the bounce and the high precision we need, one may worry that gravitational corrections can introduce a substantial uncertainty in Higgs mass bound (2.8). Fortunately, this is likely to be not the case in the most interesting range of parameters $m_H \simeq 115\,\text{GeV}$.

Indeed, in general, there are two sorts of gravitational corrections. First, there are corrections that can be reliably calculated within effective field theory. The leading correction of this kind is due to the gravitational contribution to the tree level bounce action. A recent calculation [57] provides the following handy analytic approximation for this correction

$$\Delta S_{\text{grav}} \approx \frac{1024\pi^3}{5(RM_{Pl}\lambda)^2}\,. \tag{2.10}$$

This contribution is straightforward to include in our analysis, and at the required level of precision it is negligible for Higgs masses up to $m_H \simeq 120\,\text{GeV}$ (with the appropriately shifted central values of m_t and α_s such that this Higgs mass saturates the percolation bound). For the values of the parameters such that the percolation bound is saturated at $m_H \simeq 120\,\text{GeV}$, the correction in (2.10) shifts m_H by 0.1 GeV, while for smaller m_H the correction rapidly goes to zero.

The log-divergent graviton loop contribution is also calculable and is further suppressed with respect to the tree level correction (2.10). The incalculable corrections arise due to our

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ignorance about the full theory of quantum gravity, and correspond to the power divergent loop contributions in the effective theory. They can be conveniently parameterized by the higher-dimensional operators of the form

$$\Delta\mathcal{L} = \frac{c}{M_{Pl}^2}\frac{\phi_h^6}{6!} + \dots$$

in the effective action, where ϕ_h is the Higgs field and c is a dimensionless coefficient. The effect of these operators on the bounce action (2.5) can be approximated as

$$\Delta S'_{\rm grav} \sim \frac{cR^4}{M_{Pl}^2}\phi_b^6 \sim \frac{c}{\lambda^3(RM_{Pl})^2}\,, \tag{2.11}$$

where at the last step we plugged in the value of the Higgs field in the center of the bounce $\phi_b \sim \lambda^{-1/2}R^{-1}$.

The presence of such a correction is equivalent to the additional uncertainty in the determination of the quartic coupling at the bounce scale of order

$$\frac{\delta\lambda}{\lambda} \simeq \frac{c}{\lambda^2(RM_{Pl})^2}\,. \tag{2.12}$$

Note now that the uncertainty in the quartic coupling λ at high scales can be related to the shift in the prediction for the Higgs mass as

$$\frac{\delta m_H}{m_H} = \frac{1}{2}\frac{\delta\lambda}{\lambda}(\mu = m_H) \sim \frac{\delta\lambda}{\lambda}(\mu = 1/R)\,. \tag{2.13}$$

Since we need a precision of order $\lesssim 0.2\,\mathrm{GeV}$ on the Higgs mass, we need $\delta\lambda/\lambda(\mu = 1/R) \lesssim 10^{-3}$.

At the effective field theory level there is no reason to expect this correction to be large unless the bounce energy-density, which is of order $\lambda^{-1}R^{-4}$, reaches the Planck scale. This expectation is in agreement with (2.12) if $c \geq \mathcal{O}(\lambda^{3/2})$ and, indeed, diagrams present in the effective field theory generate operators whose contributions are at most of order of the one in (2.11) with $c = \mathcal{O}(\lambda^{3/2})$. This kind of corrections are always smaller than the calculable one (2.10) and are never important in the interesting range of parameters.

Note, however, that in quantum gravity, in particular in string theory, one may expect larger values of c. Indeed, at the perturbative level the reason why c is small for small λ is that it is protected by the shift symmetry that gets restored at $\lambda = 0$. However, in quantum gravity global symmetries are always broken and, in principle, one may have even $c \simeq 1$. For instance, in the context of string theory, scalar fields usually have some geometrical meaning, so that the size of higher order corrections is typically controlled by fields vev's in Planck or string units rather than just by the energy density.

In the worst case scenario $c \simeq 1$ the incalculable correction (2.11) is enhanced by a factor $1/\lambda$ with respect to the tree level contribution (2.10). As a result it gives a shift of $0.1\,\mathrm{GeV}$ if the parameters are such that the percolation bound is saturated at $m_H \simeq 117\,\mathrm{GeV}$, as before, for smaller m_H the shift decays quickly.

We see that even under the most pessimistic assumptions gravitational corrections are only marginally important in the interesting parameter range. Still, we can take a

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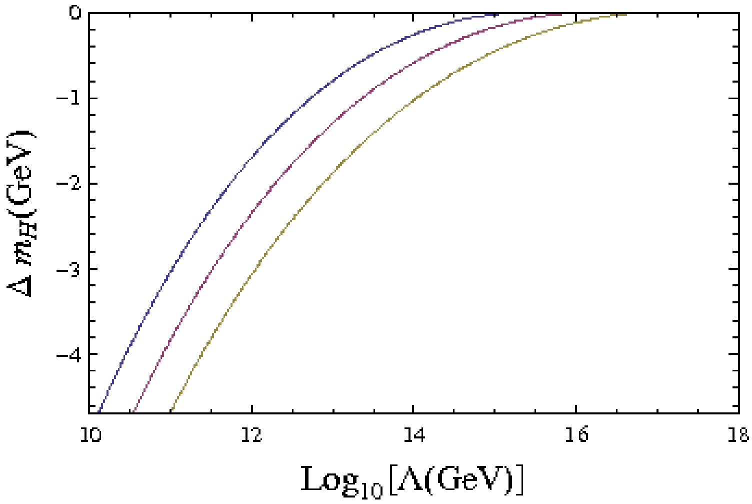

Figure 3: Decrease of the upper bound on the Higgs mass in eq. (2.8) from lowering the cutoff Λ for various values of the top mass (from the left $m_t = 171$, 173 and 175 GeV).

conservative attitude and lower the maximum scale at which the calculation should be trusted. As we lower the maximum UV scale, the decay rate decreases, and therefore the percolation bound on the Higgs mass (2.8) goes down. In figure (3) we show how the bound on the Higgs mass decreases as we decrease the cutoff Λ of our calculation for various top masses. We see that imposing the cutoff to be as small as $\sim 10^{16}$ GeV, so that even the worst case scenario gravitational corrections can be safely ignored, amounts to decreasing the upper bound on the Higgs mass only by a tiny amount. We further see that the bound on the Higgs mass does not get very much decreased (only by a few GeV) by imposing the cutoff to be as small as 10^{12} GeV, a value which allows to accommodate GUT and seesaw.

Related to this point, one may wonder what would be the implications if in this scenario the Higgs happened to be much lighter (of the order of a few GeV) than the bound of (2.8). Given that the percolation bound is very close to the life-time bound (2.6), in order to avoid the conclusion that our probability to survive up to now is ridiculously small, as it is very well known, one is forced to conclude that the scale of new physics is rather low, see e.g. [30, 32]. With this in mind, figure (3) would tell us the scale of new physics.

2.1 Vacuum decay in the MSSM

Although it is possible that the Standard Model is valid up to high scales, it is not natural from an effective field theory point of view. It is highly expected that new physics must enter at the TeV scale to cancel quadratic divergences in the Higgs potential and stabilize the electroweak scale. The most promising candidate appears to be supersymmetry, with the MSSM being its simplest realization. If supersymmetry (or any other new physics) is discovered at the LHC the analysis performed in the previous section no longer holds, since the Standard Model will stop to be a good description way before the quartic Higgs coupling

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can become negative. In particular in the MSSM the quartic terms in the Higgs potential are provided by D terms that, being proportional to the squared gauge couplings, never turn negative. Nevertheless this does not rule out all the possibilities to test in forthcoming accelerators whether the current stage of inflation may be non-eternal. Indeed, the scalar potential of the MSSM includes also squark fields, and for certain choices of soft terms, non-trivial vacua may form. These correspond to non-trivial vev's for the squarks, which break charge and color.

In this framework the situation is even more interesting than in the case of the Standard Model since the potential develops, already at tree level, new vacua at the TeV scale, thus their existence does not rely on assumptions about UV physics. As for the Standard Model, the condition not to have eternal inflation, together with the requirement to have an enough long-lived Universe allows only very specific combinations of parameters. However, the presence of many parameters entering the expression of the scalar potential makes the actual constraint very model dependent. In [58] the authors studied the tunneling rate for different choices of the MSSM parameters; they identified an "empirical region" of the parameter space with a high probability to encounter a metastable SM-like vacuum, long-lived enough to allow the Universe to survive until today. The condition reads

$$3 < \frac{A_t^2 + 3\mu^2}{m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2} < 7.5\,, \tag{2.14}$$

where A_t, μ, $m_{\tilde{t}_{L/R}}$ are the top A-term, the μ-term and the stop masses. At the lower bound [59] the Universe becomes metastable, while at the upper bound the decay rate starts being faster than the inverse life time of the Universe. Since the percolation transition occurs when $\Gamma \approx H_\Lambda^4$, a decay that avoids eternal inflation is expected close to the upper region of the interval (2.14), i.e. when

$$A_t^2 + 3\mu^2 \simeq 7.5(m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2)\,. \tag{2.15}$$

As before, this is mainly due the exponential dependence of the rate on the parameters of the action and because the window in the decay rate between percolation and the observed lifetime of the Universe is very narrow. Observing MSSM parameters to satisfy relation (2.15) may be interpreted as a strong indication that the current acceleration of the Universe is not eternal. We should say, however, that for particular choices of the MSSM parameters, vacuum decays that avoid the eternal inflation phase may happen also away from the condition (2.15). Indeed, as discussed in [58], the bound in eq. (2.14) is neither a necessary nor a sufficient condition for metastability. It is still true however, that the combinations of parameters allowing fast enough decays are a minuscule fraction of the parameter space so that, measuring the MSSM parameters in those regions would enforce the case for a non-eternal acceleration of our Universe.

Since non-eternal inflation bounds give very sharp constraints, in order to test them an equally high precision is needed in the determination of the parameters, as discussed in the previous section for the Standard Model. Indeed the bounce action at the percolation point is expected to be ~ 400 [58], (slightly less than in the SM case since the size of the

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bubble here is larger) which translates into better than one percent accuracy in the scalar potential parameters. However, unlike the SM case, the parameters of the MSSM are very unlikely to be measured with the desired precision in the forthcoming experiments; still, if the experiments favor such scenario, it is of great importance to nail down errors as much as we can, to increase (or dump) our confidence on such interpretation.

Finally we would like to recall that, just as in the Standard Model case, this type of bounds only works in one direction: measuring the parameters in a region that produce a small decay rate (with respect to εH_Λ^4), does not rule out the possibility that new physics (at any scale) may still trigger new fast decay channels of our vacuum, thus avoiding eternal inflation.

3. A "No eternal inflation" principle?

We see that the precision measurements of the Higgs mass and other Standard Model parameters may provide a crucial information about our future. If no new physics is found at the LHC and the Higgs mass comes out to be in the $\sim 0.2\,\mathrm{GeV}$ interval such that our existence now is not very improbable, but the percolation transition takes place, this will be a very strong indication that the current state of the Universe is totally unstable. Eventually (and actually quite fast — within the time of order the current age of the Universe, $\sim 10^{10}$ yr) no space-time region able to accommodate life forms even vaguely resembling ours will exist. By itself, this would definitely be an interesting and useful thing to know.

However, given that the possibility of the percolation transition requires a detailed tuning of the Standard Model parameters and there is no anthropic reason for the Higgs mass to be below the percolation value, one may wonder whether this numerical coincidence would also indicate the existence of some dynamical mechanism operating in the underlying microscopic theory. Here we outline a line of thinking suggesting that underlying fundamental physics must make eternal inflation impossible.

The starting point is that making sense of quantum gravity in de Sitter space has proved to be a notoriously hard and confusing problem. It is widely believed that no consistent theory of pure de Sitter space is possible (although an alternative viewpoint is also being developed [60 – 62]). One piece of evidence supporting this is related to the impossibility to define the boundary observables which can be measured by a single observer in de Sitter space. Recall, that quantum theory involving dynamical gravity does not allow sharply defined local observables, so that the only solid set of data in such a theory is related to the asymptotic quantities, such as S-matrix elements in flat space, or boundary CFT correlators in AdS. The non-trivial causal structure of de Sitter does not allow to define the appropriate set of boundary observables.

A further piece of evidence comes from the properties of the Coleman-De Luccia instanton [14]. One can try to realize the eternal de Sitter space by starting with a potential with a global Minkowski or AdS minimum and possessing a local positive energy minimum. Then, a region of space filled with the scalar field sitting in the local minimum is stable under small fluctuations, but non-perturbative creation of the true vacuum bubbles causes

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it to decay. In field theory one can consider a limit when the height and/or width of the barrier separating the two vacua grows indefinitely. In this limit the bubble nucleation rate vanishes and one smoothly approaches a theory with a stable positive energy vacuum. With dynamical gravity turned on, the remarkable property of the Coleman-De Luccia instanton describing this transition is that independently of the height of the barrier its action never drops below the Poincaré recurrence time $e^{-S_{\rm dS}}$, where $S_{\rm dS} \sim (M_{Pl}/H)^2$ is the entropy of de Sitter vacuum. Sometime (for instance, in the limit of a very broad barrier) the Coleman-De Luccia instanton ceases to exist, but this does not imply that the instability rate may drop below $e^{-S_{\rm dS}}$. Instead, in this case the decay goes through the Hawking-Moss instanton [63], which physically describes a thermal jump of the field onto the top of the barrier. The fact that it is impossible to increase the life-time of metastable de Sitter phase indefinitely, strongly suggests that pure de Sitter does not exist. This conclusion is further supported by string theory where de Sitter phases are always metastable.

If one adopts the point of view that de Sitter space always corresponds to a metastable state, then the conventional logic implies that all physical information about this state is encoded in the S-matrix elements (boundary correlators) of the underlying stable Minkowski (AdS) vacuum. More specifically, in the above field theory example one may consider a process where, for instance, a large number of soft scalar quanta in the true vacuum collide and produce a big bubble of the false vacuum. Later this bubble shrinks (or, if its size is really huge, bubbles of the true vacua are produced non-perturbatively within it) and decays again in a large number of scalar quanta around the true vacuum. If in addition a number of hard quanta is added to the scattering process, the corresponding amplitude can be conveniently approximated by solving the field equations for the hard quanta in the classical soft background. By considering all possible matrix elements of this sort one can reconstruct all physical properties of the false vacuum; in fact these matrix elements provide the only unambiguous way to define what one understands by "false vacuum".

However, applying this logic in the presence of gravity one again faces a problem. Indeed, Guth and Farhi have shown [64, 65] that, at least at the classical level, it is not possible to start with a non-singular perturbation around the flat vacuum and create a bubble of the false vacuum containing more than one Hubble volume of de Sitter space. Whenever one attempts to create an inflating bubble of the false vacuum its interior collapses and the bubble turns into a black hole instead of creating exponentially inflating Universe inside.

Nevertheless, one might proceed by assuming that at the quantum level it may be possible to get around the classical obstacle preventing to form a bubble of an inflating Universe in a collision of excitations around stable vacuum. This was actually done in the original Guth-Farhi paper who suggested a singular instanton solution describing such a process. This assumption is not enough, however, to allow the extraction of de Sitter properties from scattering matrix elements. Indeed, to achieve this, one also need to end up in an out-state, *i.e* the bubble should again decay into a set of small perturbations around the true vacuum. However, if the bubble interior corresponds to an eternally inflating Universe (as is the case, for instance, of the Guth-Farhi instanton), the causal structure at late times does not resemble that of the flat space even remotely.

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A more detailed and quantitative discussion of these issues can be found in [66], where the creation of inflating bubbles is addressed in AdS space. In this case AdS/CFT allows to formulate the problem even more sharply, as the creation of an inflating bubble would correspond to a transition from a pure to a mixed state in the dual CFT, which is impossible. A somewhat different attempt to construct an S-matrix description of de Sitter space using the Lorentzian Coleman-De Luccia solution [67] was also found to be problematic due to the rapid instabilities plaguing this solution [68].

All of these issues would be resolved in a straightforward though brutal way if inflation inside the bubble were not eternal, i.e., if a consistent theory of gravity were not able to support the metastable vacua with decay rates below the critical value (or the inflationary potentials supporting eternal inflation). Indeed such a possibility has already been proposed by Page [69] to avoid another very confusing feature of eternal inflation — the proliferation of Boltzmann brains in a global picture of the spacetime. It well may be that this idea is too radical — it definitely requires conspiratorial gravitational restrictions on particle physics, although it is intriguing that the bubble size in the SM case is not that far from the Planck scale, so that gravitational corrections may start being important.

Although it seems more plausible that the eternal inflation does make sense and we have to learn how to deal with it, at the moment this “no eternal inflation” proposal does not contradict any experimental or observational data. On the theoretical side, no realization of the slow roll eternal inflation has been found in string theory so far, and the available constructions of the metastable de Sitter vacua are too approximate to definitely exclude the possibility of non-perturbative processes giving rise to faster than critical decay rates. On the observational side it does not preclude sixty e-foldings of slow roll inflation, and as we saw, might acquire a dramatic support from the future particle physics data.

Let us make a last comment. Eternal inflation is often considered as a necessary ingredient for the environmental solution to the cosmological constant problem. Indeed, without eternal inflation, it appears problematic to efficiently populate the landscape of vacua. However, we would like to stress that it is the mere existence of a vast landscape that makes it possible to find vacua with un-naturally small values of the cosmological constant and perhaps other parameters. How the vacua are realized in nature is a separate issue. We do not know whether eternal inflation is the only solution; actually, we are not even sure whether the question of “population” is well-defined: it may well be that even with eternal inflation, the naive semiclassical picture of how the landscape gets populated is misleading (see e.g. [13]). The “no eternal inflation” principle thus neither conflicts nor supports the landscape approach to the cosmological constant problem.

4. Discussion

To summarize, forthcoming particle physics data may provide a surprising twist for fundamental physics and cosmology, even in the “nightmare scenario” where nothing but a light Higgs is observed at the LHC. As has long been known, our vacuum may be metastable for a light enough Higgs. For a very narrow window of parameters, our Universe has not yet decayed but the current inflationary period can not be future eternal. To support

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this conclusion one would need to measure the Standard Model parameters with extreme precision, requiring significant but achievable progress in the theoretical calculations and experimental measurements.

As a benchmark value for the desired accuracy we used 0.2 GeV for the Higgs mass. Given the relation (2.8) this corresponds to the precision at the level of 60 MeV for the top mass (as opposed to the present $\sim 1.8\,\text{GeV}$) , and at the level of 0.14% for the strong coupling (as opposed to the current precision at the level $\sim 1.7\%$). With this precision one would be able to conclusively establish that the current stage of the cosmological acceleration is not eternal if the decay rate of our vacuum is fast enough, so that the probability that we did not decay up to now is 5%. Even higher precision is needed if the decay rate is slower. Note that in this case, even if we are not able to resolve whether the Higgs mass lies above or below the percolation bound, finding the Higgs within 0.2 GeV from that bound would be remarkable enough to take seriously the idea that eternal inflation may be impossible. This may be true for the heavier Higgs masses, when the incalculable gravitational corrections may in principle affect the bound at this level of precision.

For the Higgs mass itself the desired level of accuracy is likely to be achievable directly at the LHC [56]. Indeed, for a light Higgs, a very precise determination of the mass is possible by measuring the location of the maximum of the photon distribution from the $H \to \gamma\gamma$ decays. It is important that we need the mass itself; much higher statistics would be needed to determine the width with the same precision.

The top mass precision will not improve much at the LHC [56, 70]. The statistical uncertainties will be reduced quite significantly and will be practically negligible. The problem is due to the large systematic uncertainties from the jet energy scale and final state radiation. The best one can hope for is $\Delta m_t \sim 1\,\text{GeV}$. However, the situation improves a lot at a linear collider [71, 72]. Here one can use the shape of the cross-section for $t\bar{t}$ production near threshold to determine the top mass. This method is analogous to the extraction of the bottom mass from the spectrum of the bottomonium. At the moment the theoretical uncertainty from this method has been reduced to the level $\sim 80\,\text{MeV}$ and may improve somewhat in the future.

The best determination of α_s at the moment (and apparently in the reasonable future as well) is from lattice QCD. It appears reasonable to expect an order of magnitude improvement (i.e., the required level) within the time-scale of constructing a new linear collider.

Finally, on the theoretical side one will need to improve both the running and the matching conditions for gauge couplings and masses by at least one extra electroweak loop and two strong loops. This also requires to include another loop for the calculation of the bounce action.

It is worth mentioning that there is a theoretically motivated natural level of precision, that one may wish to achieve. Indeed, even if the Universe is eternally inflating, bubbles of new vacuum may form infinite volume clusters if the decay rate is fast enough. The exact value of $\varepsilon \equiv \varepsilon_p$ when this transition (usually referred to as the "percolation transition", contrary the terminology adopted in the current paper) happens is currently unknown and is limited to be in the range (2.3). Most likely ε_p is quite close to the upper boundary

$\varepsilon_p \sim 0.24$. It appears natural to try to achieve an accuracy allowing to distinguish this critical value ε_p from $\varepsilon = 9/4\pi$ (corresponding to the transition to the eternal regime). If $\varepsilon_p = 0.24$ this would require the Higgs mass resolution at the level of 0.05 GeV, but may be somewhat less challenging if ε_p is smaller.

There are other scenarios motivating the Higgs mass to be in the proximity of the life-time bound. For instance, a proposal of ref. [73] is based on specific assumptions on how the Standard Model parameters scan in the landscape. More generally, this may be a natural expectation if one adopts the "living dangerously" logic [8, 74].

The key observation of the current paper is the existence of a sharp critical value of the Higgs mass, having a direct physical meaning independently of any assumptions on the landscape statistics and on what one understands by being "close" to the life-time bound. A high precision is of crucial importance to take the full advantage of the existence of this sharp number. One of the benefits of having a sharp prediction, is that although we will never be able to directly test that there is no new physics up to the GUT scale, finding the Higgs mass in the correct window would provide a strong case for such a scenario.There are also very well-motivated ideas for physics at the TeV scale, such as low-energy SUSY with large A terms, where we can conclude that our vacuum is metastable. In this case high-precision measurements can in principle again tell us that the instability rate is subcritical for eternal inflation, and in this case the conclusion can be drawn much more sharply since the physics of the instability involves no further theoretical extrapolation and can be studied at the TeV scale.

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A. Details of the calculation

In this appendix we give some details of the calculation of the decay rate of the Standard Model vacuum, and provide the most relevant formulæ.

We need only the Higgs quartic λ, the gauge g, g', g_3, and top Yukawa h couplings, and work in the $\overline{\text{MS}}$ scheme. The normalization of the Higgs quartic coupling is chosen so that the tree-level potential for the physical Higgs ϕ_h reads

$$V(\phi_h) = \frac{1}{24}\lambda\left(\phi_h^2 - v^2\right)^2 , \tag{A.1}$$

where $v = (\sqrt{2}G_\mu)^{-1/2} = 246.221$ GeV and $G_\mu = 1.16637 \cdot 10^{-5}$ GeV^{-2} is the Fermi constant from muon decays. As explained in the main part of the text, we solve the RG equations for the running couplings to $\mathcal{O}(\alpha_s^3)$, $\mathcal{O}(\alpha_W^2)$, $\mathcal{O}(\alpha_s\alpha_W)$, which means two-loop plus pure α_s three-loop. For consistency, we match the initial conditions for the top Yukawa and the Higgs quartic coupling up to order $\mathcal{O}(\alpha_W)$ (one loop), and $\mathcal{O}(\alpha_s^2)$ (pure two loops in α_s).

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Higher order corrections to the β-functions and to the matching conditions (four loops in α_s to the strong coupling running, three loops in α_s and two loops mixed strong/weak to the top Yukawa matching) are known. These form an incomplete list of the quantities required for doing a consistent next order computation, and therefore we use them to estimate the theoretical errors. In order to reduce the theoretical uncertainty associated with the available expressions for the matching conditions, we perform the matching respectively for the top Yukawa at the top mass scale, for the Higgs quartic at the Higgs mass scale, and for the gauge couplings at the m_Z scale.

We are now ready to give the expression for the relevant equations. The RG equations are given by

$$\begin{aligned}
\frac{d}{dt}\lambda(t) &= \kappa\beta_\lambda^{(1)} + \kappa^2\beta_\lambda^{(2)}\,, \\
\frac{d}{dt}h(t) &= \kappa\beta_h^{(1)} + \kappa^2\beta_h^{(2)} + \kappa^3\beta_h^{(3)}\,, \\
\frac{d}{dt}g(t) &= \kappa\beta_g^{(1)} + \kappa^2\beta_g^{(2)}\,, \\
\frac{d}{dt}g'(t) &= \kappa\beta_{g'}^{(1)} + \kappa^2\beta_{g'}^{(2)}\,, \\
\frac{d}{dt}g_3(t) &= \kappa\beta_{g_3}^{(1)} + \kappa^2\beta_{g_3}^{(2)} + \kappa^3\beta_{g_3}^{(3)} + \kappa^4\beta_{g_3}^{(4)}\,,
\end{aligned} \tag{A.2}$$

where $t = \log(\mu/m_Z)$ with μ being the renormalization scale. The apex on the β-functions represents the loop order. They are given by [27, 40 – 43, 49, 50]:

$$\begin{aligned}
\beta_\lambda^{(1)} &= \frac{27}{4}g(t)^4 + \frac{9}{2}g'(t)^2g(t)^2 - 9\lambda(t)g(t)^2 + \frac{9}{4}g'(t)^4 - 36h(t)^4 + 4\lambda(t)^2 - 3g'(t)^2\lambda(t) \\
&\quad +12h(t)^2\lambda(t)\,, \\
\beta_h^{(1)} &= \frac{9}{2}h(t)^3 - \frac{9}{4}g(t)^2h(t) - 8g_3(t)^2h(t) - \frac{17}{12}g'(t)^2h(t)\,, \\
\beta_g^{(1)} &= -\frac{19}{6}g(t)^3\,, \\
\beta_{g'}^{(1)} &= \frac{41}{6}g'(t)^3\,, \\
\beta_{g_3}^{(1)} &= -7g_3(t)^3\,, \\
\beta_\lambda^{(2)} &= 80g_3(t)^2h(t)^2\lambda(t) - 192g_3(t)^2h(t)^4 + \frac{915}{8}g(t)^6 - \frac{289}{8}g'(t)^2g(t)^4 - \frac{27}{2}h(t)^2g(t)^4 \\
&\quad -\frac{73}{8}\lambda(t)g(t)^4 - \frac{559}{8}g'(t)^4g(t)^2 + 63g'(t)^2h(t)^2g(t)^2 + \frac{39}{4}g'(t)^2\lambda(t)g(t)^2 - 3h(t)^4\lambda(t) \\
&\quad +\frac{45}{2}h(t)^2\lambda(t)g(t)^2 - \frac{379}{8}g'(t)^6 + 180h(t)^6 - 16g'(t)^2h(t)^4 - \frac{26}{3}\lambda(t)^3 - \frac{57}{2}g'(t)^4h(t)^2 \\
&\quad -24h(t)^2\lambda(t)^2 + 6\left(3g(t)^2 + g'(t)^2\right)\lambda(t)^2 + \frac{629}{24}g'(t)^4\lambda(t) + \frac{85}{6}g'(t)^2h(t)^2\lambda(t)\,, \\
\beta_h^{(2)} &= h(t)\Bigg[-108g_3(t)^4 + 9g(t)^2g_3(t)^2 + \frac{19}{9}g'(t)^2g_3(t)^2 + 36h(t)^2g_3(t)^2 - \frac{3}{4}g'(t)^2g(t)^2 \\
&\quad -\frac{23}{4}g(t)^4 + \frac{1187}{216}g'(t)^4 - 12h(t)^4 + \frac{\lambda(t)^2}{6} + h(t)^2\left(\frac{225}{16}g(t)^2 + \frac{131}{16}g'(t)^2 - 2\lambda(t)\right)\Bigg]\,,
\end{aligned} \tag{A.3}$$

$$
\begin{aligned}
\beta_g^{(2)} &= 12 g_3(t)^2 g(t)^3 + \left(\frac{35}{6} g(t)^2 + \frac{3}{2} g'(t)^2 - \frac{3}{2} h(t)^2\right) g(t)^3 \,, \\
\beta_{g'}^{(2)} &= \frac{44}{3} g_3(t)^2 g'(t)^3 + \left(\frac{9}{2} g(t)^2 + \frac{199}{18} g'(t)^2 - \frac{17}{6} h(t)^2\right) g'(t)^3 \,, \\
\beta_{g_3}^{(2)} &= g_3(t)^3 \left(\frac{9}{2} g(t)^2 - 26 g_3(t)^2 + \frac{11}{6} g'(t)^2 - 2h(t)^2\right) \,, \\
\beta_h^{(3)} &= 384 g_3(t)^6 \left(-\frac{2083}{576} + \frac{5}{3}\zeta_3\right) \,, \\
\beta_{g_3}^{(3)} &= \frac{65}{2} g_3(t)^7 \,, \\
\beta_{g_3}^{(4)} &= g_3(t)^9 \left(\frac{63559}{18} - \frac{44948}{9}\zeta_3\right) \,.
\end{aligned}
$$

where $\zeta_3 = 1.20206\ldots$ is the Riemann zeta function and

$$
\kappa \equiv \frac{1}{16\pi^2} \,. \tag{A.4}
$$

Notice that the 4-loop QCD correction to the β-function of g_3 ($\beta_{g_3}^{(4)}$) is an higher order effect and has only been used to estimate the theoretical uncertainty. The matching between the top pole mass and the $\overline{\mathrm{MS}}$ Yukawa is given by

$$
h(\mu) = 2^{3/4}\sqrt{G_\mu}\, m_t \Big(1 + \delta_t(\mu)\Big) \,, \tag{A.5}
$$

where

$$
\delta_t(\mu) = \delta_t^{\mathrm{QCD}}(\mu) + \delta_t^{W}(\mu) + \delta_t^{\mathrm{QED}}(\mu) \,. \tag{A.6}
$$

Here $\delta_t^W + \delta_t^{\mathrm{QED}}$ represent the one loop electroweak contribution and is given by [45]

$$
\delta_t^W(\mu) + \delta_t^{\mathrm{QED}}(\mu) = -\frac{E(\mu)}{2} + \mathrm{Re}\Big[\Sigma_V(\mu) + \Sigma_S(\mu)\Big] - \frac{\Pi(\mu)}{2m_W^2} \,, \tag{A.7}
$$

where

$$
E(\mu) = \frac{\alpha_{em}(m_Z)}{4\pi s^2}\left[\left(\frac{7}{2s^2} - 6\right)\log\left(c^2\right) - 4\log\left(\frac{m_Z^2}{\mu^2}\right) + 6\right] \,, \tag{A.8}
$$

$$
\begin{aligned}
\Sigma_V(\mu) + \Sigma_S(\mu) = {} & \frac{\alpha_{em}(m_Z)}{4\pi}\Bigg\{\left(6 - \frac{m_Z^2}{m_t^2}\right) a_f^2 - 4Q_t^2 - \left(\frac{m_Z^2}{m_t^2} + 4\right) v_f^2 \qquad \text{(A.9)} \\
& + \left[a_f^2\left(4 - \frac{m_Z^2}{m_t^2}\right) - \left(\frac{m_Z^2}{m_t^2} + 2\right) v_f^2\right] F\left(m_t^2, m_t^2, m_Z^2\right) \\
& - \left[\frac{3}{8s^2}\left(\frac{m_t^2 - m_b^2}{m_W^2} + 1\right) - 3\left(Q_t^2 + v_f^2 - a_f^2\right) + \frac{1}{8c^2}\right]\log\left(\frac{m_t^2}{\mu^2}\right) \\
& + \frac{1}{4m_W^2 s^2}\Bigg[4m_t^2 - \frac{5m_b^2}{2} + \frac{1}{2}\left(m_W^2 - m_H^2\right) + \frac{m_b^4 + m_b^2 m_W^2 - 2m_W^4}{2m_t^2} \\
& + \frac{1}{2m_t^2}\left(\left(m_b^2 + m_t^2\right) m_W^2 + \left(m_t^2 - m_b^2\right)^2 - 2m_W^4\right) F\left(m_t^2, m_b^2, m_W^2\right)
\end{aligned}
$$

$$+\left(2m_t^2-\frac{m_H^2}{2}\right)F\left(m_t^2,m_t^2,m_H^2\right)+\left(m_W^2+\frac{1}{2}\left(m_t^2-3m_b^2\right)\right)\log\left(\frac{m_t^2}{m_b^2}\right)$$
$$+\frac{1}{2}m_H^2\left(3-\frac{m_H^2}{2m_t^2}\right)\log\left(\frac{m_t^2}{m_H^2}\right)+\frac{1}{4m_t^4}\Big(3\left(m_b^2+m_t^2\right)m_W^4+4m_t^4m_W^2$$
$$+\left(m_t^2-m_b^2\right)^3-2m_W^6\Big)\log\left(\frac{m_b^2}{m_W^2}\right)\Bigg]$$
$$+\left[a_f^2\left(2-\frac{m_Z^4}{2m_t^4}+\frac{3m_Z^2}{m_t^2}\right)-\frac{m_Z^4v_f^2}{2m_t^4}\right]\log\left(\frac{m_t^2}{m_Z^2}\right)\Bigg\}\,,$$

$$\Pi(\mu)=\frac{\alpha_{em}(m_Z)m_W^2}{4\pi s^2}\left[\frac{7}{8c^2}-\frac{17}{4}-\frac{3m_H^2}{4\left(m_W^2-m_H^2\right)}\log\left(\frac{m_W^2}{m_H^2}\right)-\frac{m_H^2}{8m_W^2}\right. \tag{A.10}$$
$$\left.+\left(2+\frac{1}{c^2}-\frac{17}{4s^2}\right)\log\left(c^2\right)-\left(\frac{1}{c^2}-2\right)\log\left(\frac{m_W^2}{\mu^2}\right)\right]$$
$$+\frac{3\alpha_{em}(m_Z)}{4\pi s^2}\left[\frac{m_b^2m_t^2}{m_t^2-m_b^2}\log\left(\frac{m_t^2}{m_b^2}\right)-\left(\frac{1}{2}-\log\left(\frac{m_b^2}{\mu^2}\right)\right)m_b^2\right.$$
$$\left.-m_t^2\left(\frac{1}{2}-\log\left(\frac{m_t^2}{\mu^2}\right)\right)\right]\,,$$

with

$$F(x,y,z)=\begin{cases}\dfrac{\tilde\lambda(x,y,z)^{\frac{1}{2}}}{x}\,\mathrm{arccosh}\left(\dfrac{-x+y+z}{2\sqrt{yz}}\right) & \text{if } x<\left(\sqrt{y}-\sqrt{z}\right)^2\\[2ex] -\dfrac{\sqrt{-\tilde\lambda(x,y,z)}}{x}\,\arccos\left(\dfrac{-x+y+z}{2\sqrt{yz}}\right) & \text{if } \left(\sqrt{y}-\sqrt{z}\right)^2\le x\le\left(\sqrt{y}+\sqrt{z}\right)^2\\[2ex] \dfrac{\tilde\lambda(x,y,z)^{\frac{1}{2}}}{x}\left[i\pi-\mathrm{arccosh}\left(\dfrac{x-y-z}{2\sqrt{yz}}\right)\right] & \text{if } x>\left(\sqrt{y}+\sqrt{z}\right)^2\,,\end{cases} \tag{A.11}$$

and

$$\tilde\lambda(x,y,z)=x^2+y^2+z^2-2(xy+zy+xz)\,, \tag{A.12}$$

$$s=\sqrt{1-\frac{m_W^2}{m_Z^2}},\qquad c=\frac{m_W}{m_Z},\quad Q_t=\frac{2}{3},\qquad v_f=\frac{1}{4sc}-\frac{2s}{3c},\qquad a_f=\frac{1}{4sc}\,. \tag{A.13}$$

The numerical values we used are [54]

$$m_b=4.2\text{ GeV},\quad m_Z=91.1876\text{ GeV},\quad m_W=80.403\text{ GeV},\quad \alpha_{em}^{-1}(m_Z)=127.9\,. \tag{A.14}$$

Concerning the pure QCD contribution to δ_t, we have the one, two and three loop results [46 – 48]

$$\delta_t^{\mathrm{QCD},(1)}(\mu)=-\frac{4\alpha_s(\mu)}{3\pi}\left(1-\frac{3}{4}\log\left(\frac{m_t^2}{\mu^2}\right)\right)\,, \tag{A.15}$$

$$\delta_t^{\mathrm{QCD},(2)}(m_t)=(-14.3323+1.0414\times 5)\left(\frac{\alpha_s(m_t)}{\pi}\right)^2\,, \tag{A.16}$$

$$\delta_t^{\mathrm{QCD},(3)}(m_t) = (-0.65269 \times 5^2 + 26.9239 \times 5 - 198.7068) \left(\frac{\alpha_s(m_t)}{\pi}\right)^3 . \quad \text{(A.17)}$$

Notice that the pure 2-loop and 3-loop contributions are known only at the top mass scale, and this is why we perform the matching for the top at this scale. We do not use $\delta_t^{\mathrm{QCD},(3)}$ directly in deriving our bound (2.8), but just to estimate the theoretical uncertainty. Further, the mixed 2-loop strong/weak matching correction is known [52], and it is numerically comparable to the effect of (A.17).

Turning to the Higgs, we match the Higgs pole mass to the $\overline{\mathrm{MS}}$ quartic coupling at one loop through the following [44]

$$\lambda(\mu) = 3\sqrt{2} G_\mu m_H^2 \Big(1 + \delta_H(\mu)\Big) , \quad \text{(A.18)}$$

where

$$\delta_H(\mu) = \frac{G_\mu m_Z^2}{8\sqrt{2}\pi^2} \left(\xi f_1(\mu) + f_0(\mu) + \frac{f_{-1}(\mu)}{\xi}\right) , \quad \text{(A.19)}$$

with

$$\begin{aligned}
\xi &= \frac{m_H^2}{m_Z^2} , \\
f_1(\mu) &= \frac{3}{2}\log(\xi) - \log\left(c^2\right) + 6\log\left(\frac{\mu^2}{m_H^2}\right) - \frac{1}{2} Z\left[\frac{1}{\xi}\right] - Z\left[\frac{c^2}{\xi}\right] + \frac{9}{2}\left(\frac{25}{9} - \frac{\pi}{\sqrt{3}}\right) , \\
f_0(\mu) &= \frac{3c^2}{s^2}\log\left(c^2\right) + 12\log c^2\left(c^2\right) + \frac{3\xi c^2}{\xi - c^2}\log\left(\frac{\xi}{c^2}\right) + 4c^2 Z\left[\frac{c^2}{\xi}\right] - \frac{15}{2}\left(2c^2+1\right) \\
&\quad -6\left(2c^2 - \frac{2m_t^2}{m_Z^2} + 1\right)\log\left(\frac{\mu^2}{m_Z^2}\right) - \frac{3m_t^2}{m_Z^2}\left(4\log\left(\frac{m_t^2}{m_Z^2}\right) + 2\,Z\left[\frac{m_t^2}{m_Z^2\xi}\right] - 5\right) \\
&\quad +2\,Z\left[\frac{1}{\xi}\right] , \\
f_{-1}(\mu) &= 8\left(2c^4+1\right) - 12c^4\log\left(c^2\right) - 12c^4\,Z\left[\frac{c^2}{\xi}\right] + 6\left(2c^4 - \frac{4m_t^4}{m_Z^4} + 1\right)\log\left(\frac{\mu^2}{m_Z^2}\right) \\
&\quad -6\,Z\left[\frac{1}{\xi}\right] + \frac{24m_t^4}{m_Z^4}\left(\log\left(\frac{m_t^2}{m_Z^2}\right) + Z\left[\frac{m_t^2}{m_Z^2\xi}\right] - 2\right) , \\
Z[z] &= \begin{cases} 2A(z)\arctan\left(\frac{1}{A(z)}\right) & \text{if } z > \frac{1}{4} \\ A(z)\log\left(\frac{A(z)+1}{1-A(z)}\right) & \text{if } z < \frac{1}{4} \end{cases} , \\
A(z) &= \sqrt{|1-4z|} . \qquad \text{(A.20)}
\end{aligned}$$

We finally give the formula for ΔS [34]

$$\begin{aligned}
\Delta S(t) &= 6\left(6L(t)+5\right)\frac{h(t)^4}{\lambda(t)^2} + \left(12L(t)+13\right)\frac{h(t)^2}{|\lambda(t)|} - \frac{2}{3}\left(6L(t)+5\right) \\
&\quad - \left(6L(t)+7\right)\frac{2g(t)^2 + g_Z(t)^2}{2|\lambda(t)|} - 9\left(2L(t)+1\right)\frac{2g(t)^4 + g_Z(t)^4}{8\lambda(t)^2} \\
&\quad + f_h\left(\lambda(t)\right) + 2f_g\left(\frac{6g(t)^2}{|\lambda(t)|}\right) + f_g\left(\frac{6g_Z(t)^2}{|\lambda(t)|}\right) - f_t\left(\frac{6h(t)^2}{|\lambda(t)|}\right) ,
\end{aligned} \quad \text{(A.21)}$$

where

$$L(t) = \log\left(\frac{R\, m_Z\, e^{t+\gamma_E}}{2}\right), \qquad g_Z^2 = g^2 + g'^2\ . \tag{A.22}$$

Here R is the radius of the bubble, and $f_t,\ f_g,\ f_h$ are functions whose numerical values can be found plotted in [34].[3] In the numerical results given in the text the renormalization scale has been set to $\mu = 2e^{-\gamma_E}/R$, so that $L(t) = 0$; the dependence on the choice of the scale μ has been checked to be small.

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